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Sensitivity-Based Characterization and Optimization of Viscoelastically Damped Honeycomb Structures

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Control of dynamic vibrations is critical to the operational success of many aerospace systems. Large space structures, for practical reasons, are lightweight and hence highly flexible. This paper addresses the problem of vibration control of such structures through passive damping, using viscoelastic materials. A new sensitivity-based approach to characterize viscoelastic material property variation with frequency is outlined. It is demonstrated that, using structural optimization techniques, viscoelastically damped honeycomb structures can be tailored to provide weight-effective vibration damping. A spacecraft payload mounting platform is used as the demonstration structure.

Introduction

THERE are a growing number of applications for which the control of dynamic vibrations is critical to the operational success of the system. Typical applications include the control of vibrations in large space structures, airframes, and automotive suspension systems. This vibration control may be either passive or active. In a large space structure for instance, the primary mission is usually to gather/transmit/receive information from the ground, space, or other deployed satellites, and in most cases process this information onboard. Mission goals dictate stringent pointing requirements for precision instruments such as Earth observation systems, thus making dynamic stability of these structures a complex requirement. In this environment, the use of damping mechanisms is crucial to the successful operation of the satellite.

For practical reasons, very large space structures are lightweight, and hence highly flexible. They are in general characterized by low stiffness-to-mass ratios, resulting in a cluster of many low-frequency, lightly damped vibration modes. Vibration control of such systems in a weight-effective, reliable, and robust manner is one of the most challenging problems facing the structural dynamics community today.

Passive damping provides the most robust method of providing moderate levels of vibration control for large space structures. Passive damping involves either the modification of key structural design parameters which affect the structural mass, stiffness, and damping characteristics, or the use of special energy-absorbing materials, like the viscoelastic materials. Viscoelastic materials have been effectively used to control resonant vibration in space.[1] This paper considers a constrained viscoelastic layer treatment,[2] consisting of a thin layer of viscoelastic film bonded to the flexible structure's surface, followed by a stiff constraining layer bonded to the viscoelastic layer's surface. During deformation, the shearing strain induced in the viscoelastic core results in the desired energy dissipation and thereby reducing vibration.

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This paper deals with optimization techniques for weight-efficient design of viscoelastically damped structures with frequency and damping requirements. A new approach, using function sensitivities, is outlined to characterize the viscoelastic material property variation with frequency. Hybrid approximations to damping and frequency constraints are used to reduce the computational effort required for solving the nonlinear programming problem, and a mode-tracking procedure is developed to identify the right vibration modes during design optimization. Finally, these developments are demonstrated on a spacecraft payload mounting platform using the external stiffener damping concept.

Matrix Equations of Motion

The equations of motion for the finite element discretization of the damped structure is of the form

$$M\ddot{q} + C\dot{q} + Kq = P(t) \tag{1}$$

where

M, C, K = mass, damping, and stiffness matrices
$q, \dot{q}, \ddot{q}$ = nodal displacement, velocity, and acceleration vectors
$P(t)$ = vector of applied loads

Several methods like the direct method (frequency-response analysis) and the modal strain energy method can be used to solve Eq. (1) for the dynamic response. In this study, the modal strain energy method programmed in MSC/Nastran Version 66 is used.[3]

Modal Strain Energy Method

In the modal strain energy (MSE) method, a basic assumption is that the damped structure can be represented using the normal modes of the associated undamped system, provided the necessary damping terms are included in the modal equations of motion.

Using linear transformations from the physical to the modal coordinate system, the decoupled equations of motion are written as

$$\ddot{\alpha}_r + \eta^{(r)}\omega_r\dot{\alpha}_r + \omega_r^2\alpha_r = p_r(t) \tag{2}$$

$$q = \Sigma\phi^{(r)}\alpha_r(t), \qquad r = 1, 2, \ldots \tag{3}$$

where

α_r = rth modal coordinate
ω_r = natural frequency of the rth mode
$\eta^{(r)}$ = modal loss factor of the rth mode
$\phi^{(r)}$ = rth undamped mode shape

The modal loss factors are computed using the following relation[1]:

$$\eta^{(r)} = \beta^{(r)} V_v^{(r)} / V^{(r)} \tag{4}$$

where $\beta^{(r)}$ is the viscoelastic material loss factor at the rth modal frequency, $V_v^{(r)}$ is the elastic strain energy in the viscoelastic layer at the rth modal frequency, and $V^{(r)}$ is the total strain energy at the rth modal frequency.

The ratio of the elastic strain energies in Eq. (4) is computed as

$$\frac{V_v^{(r)}}{V^{(r)}} = \sum_i^{nv} \frac{\phi_i^{(r)T} K_i \phi_i^{(r)}}{\phi^{(r)T} K \phi^{(r)}} \tag{5}$$

where nv is the number of viscoelastic elements in the finite element model, and K_i is the stiffness matrix of the ith viscoelastic element. A drawback of the modal strain energy approach for damping design is that it does not take into account the frequency dependency of the viscoelastic materials.

Damping Material Behavior

Successful design of passive damping treatments using viscoelastic materials (VEMs) depends on accurate characterization of how the properties of these materials vary with temperature and frequency.[4] VEMs are generally more difficult to characterize than are materials such as metals for two primary reasons. First, VEMs convert a much larger percentage of the input dynamic energy into heat than does a metal. Consequently, it is necessary to measure both the energy storage property (stiffness) and the energy dissipation property (damping). Second, both stiffness and damping vary greatly with frequency, temperature, and in some cases, strain level. The shear modulus G and the material loss factor β of the viscoelastic material are functions of frequency and temperature[5]:

$$G = G(f,T), \qquad \beta = \beta(f,T) \tag{6}$$

Generally, the greater potential for damping in a material, the greater the variation.

The mechanical properties of VEMs are described by using a frequency- and temperature-dependent complex modulus

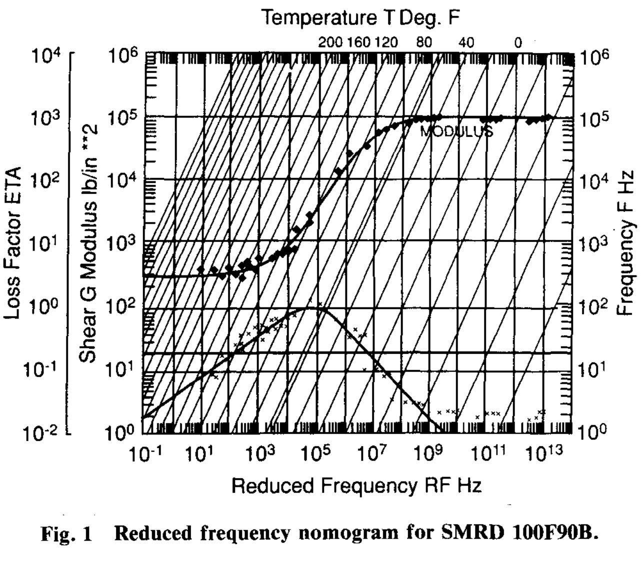

Fig. 1 Reduced frequency nomogram for SMRD 100F90B.

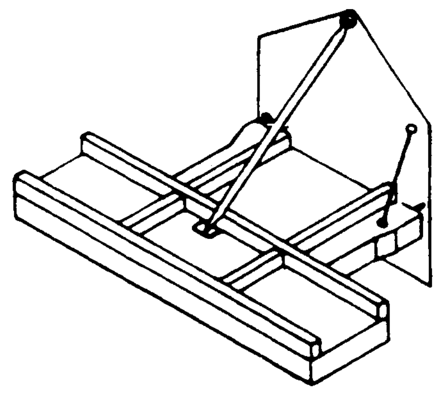

Fig. 2 Payload mounting platform.

G^*. The complex shear modulus can be expressed in the form

$$G^*(f,T) = G_0(f,T)\left[1.0 + j\beta(f,T)\right] \tag{7}$$

The real and imaginary parts of the modulus are $G_0(f,T)$ and $G_0(f,T)\beta(f,T)$, respectively. The two frequency-temperature functions on the right in Eq. (7) are commonly called the storage and loss modulus.

When characterizing a viscoelastic material, a temperature shift function, called α_T, which is a function of temperature only, is constructed for each particular set of complex modulus data. The real and the imaginary parts (G_R and G_I) of the complex viscoelastic shear modulus and the material loss factor β are plotted as a function of reduced frequency rf, where rf is the product of the experimental frequency and α_T:

$$rf = f_i \alpha_T \tag{8}$$

Historically, the temperature shift function is defined empirically by the experimental data. The value of α_T at each experimental temperature is chosen such that it simultaneously shifts the three complex modulus points (G_R, G_I, β) to define curves and minimize scatter.

A reduced-temperature nomogram, or international plot, shown in Fig. 1, is used to characterize the material based on frequency and temperature. The curve expresses the real modulus and loss factor as a function of reduced frequency. The temperature shift function is then used to superimpose lines of constant temperature between the reduced frequency and the experimental frequency which is placed on the right-hand scale. A reduced-frequency value (vertical line) is defined as the intersection of a frequency value (horizontal line) and a temperature value (diagonal line). A damping designer may then read modulus and loss factor off this plot for any particular frequency and temperature of interest.

Viscoelastic materials have a wide range of properties that depend on the frequency and temperature of interest. Peak damping occurs in the transition region between the lower storage modulus rubbery asymptote and the upper storage modulus glassy asymptote. However, for the damped structural analysis (modal strain energy method), a single value of the shear modulus G_{ref} needs to be used to obtain the modal frequencies, mode shapes, and strain energy ratios. Hence, due to the frequency-dependent nature of the viscoelastic material, each finite element analysis model will be consistent with the viscoelastic core material properties only for one mode. A simple correction to the modal loss factor $\eta^{(r)}$ (obtained from the normal modes analysis with $G = G_{\text{ref}}$) is used in Ref. 2 to accommodate the frequency dependency of the viscoelastic materials:

$$\eta^{(r)\prime} = \eta^{(r)} \sqrt{G(f_r)/G_{\text{ref}}} \tag{9}$$

where $\eta^{(r)\prime}$ is the adjusted modal loss factor for the rth mode, and $G(f_r)$ is the core shear modulus at $f = f_r$ where f_r is the rth modal frequency calculated with $G = G_{\text{ref}}$. However, when this correction factor was used on the payload mounting platform (PMP),[6] shown in Fig. 2, the adjusted modal loss factors were significantly different from the exact modal loss factors computed using $G(f_r)$ for the normal modes analysis. Hence, an alternate approach using sensitivity coefficients is proposed to account for the frequency dependency of the damping material.

Sensitivity-Based Characterization of Viscoelastic Material Property Variation with Frequency

The proposed sensitivity-based approach adjusts the loss factor using the gradients of strain energy ratio in a hybrid Taylor series linearization with respect to the frequencies. The procedure is as follows.

Let $G(\omega_1)$ correspond to the shear modulus of the viscoelastic material at the fundamental frequency of the structure ω_1. A normal modes analysis is then performed with the shear modulus of the viscoelastic equal to $G(\omega_1)$. This normal modes analysis is then followed by strain energy calculations to compute the modal loss factors $\eta^{(r)}$.

$$\eta^{(r)} = \beta^{(r)}\epsilon^{(r)}, \quad \text{and} \quad \epsilon^{(r)} = V_v^{(r)}/V^{(r)}, \quad r = 1,2,\ldots,n \tag{10}$$

It is now necessary to adjust the modal loss factors $\eta^{(r)}$, $r = 2,3,\ldots,n$, for the viscoelastic shear modulus variation with frequencies ω_r, $r = 2,3,\ldots,n$. This adjustment is done by first updating the strain energy ratios, using a hybrid Taylor series linearization, for frequencies ω_r, $r = 2,3,\ldots,n$.

$$\epsilon_\omega^{(r)\text{adj}} = \epsilon_{\omega 1}^{(r)\text{calc}} + Q_r\left[\frac{\partial\epsilon^{(r)}}{\partial G(\omega_1)}\right]\left(\frac{\partial G}{\partial\omega}\right)\delta\omega, \quad \text{for} \quad r = 2,3,\ldots,n \tag{11}$$

where $\delta\omega$ is the change in frequency $= (\omega_r - \omega_1)$, $r = 2,3,\ldots,n$; $\epsilon_{\omega 1}^{(r)\text{calc}}$ is the strain energy ratio of the rth mode, obtained from the modal strain energy analysis with shear modulus of $G(\omega_1)$; $\epsilon_\omega^{(r)\text{adj}}$ is the adjusted strain energy of the rth mode; and

$$Q_r = 1 \quad \text{if} \quad \left[\frac{\partial\epsilon^{(r)}}{\partial G(\omega_1)}\right]\left(\frac{\partial G}{\partial\omega}\right) > 0$$

$$= \omega_1/\omega_r \quad \text{if} \quad \left[\frac{\partial\epsilon^{(r)}}{\partial G(\omega_1)}\right]\left(\frac{\partial G}{\partial\omega}\right) \le 0$$

When $Q_r = 1$, the hybrid linearization is the same as a direct Taylor series expansion and otherwise reduces to the reciprocal approximation. The gradient of the shear modulus with respect to frequency is obtained from a reduced-frequency nomogram generated from test results.

Having updated the strain energy ratios, the adjusted modal loss factor is calculated as

$$\eta^{(r)\prime} = \beta^{(r)}\epsilon^{(r)\text{adj}} \tag{12}$$

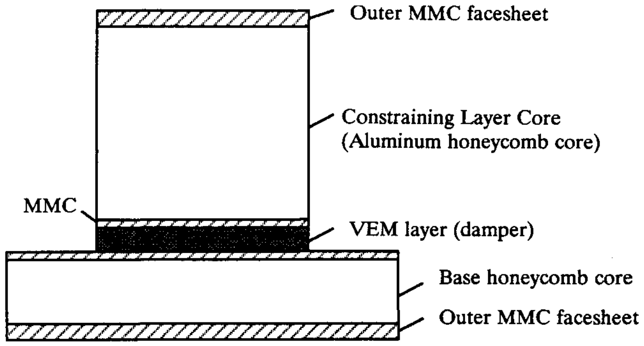

Fig. 3 Three-layered sandwich structure (cross section).

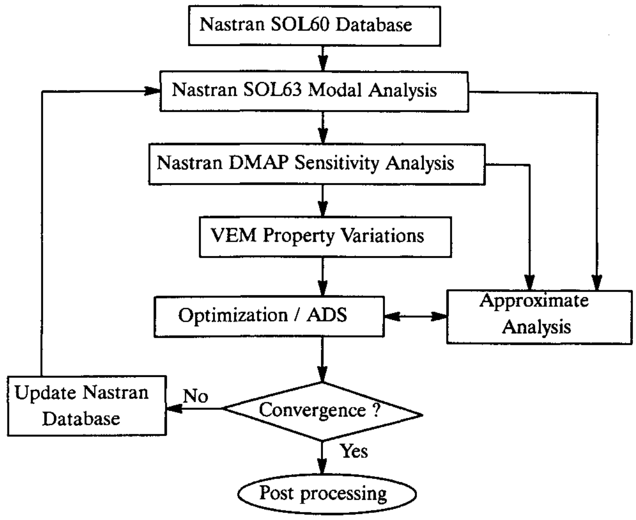

Fig. 4 DAMP-OPT system architecture.

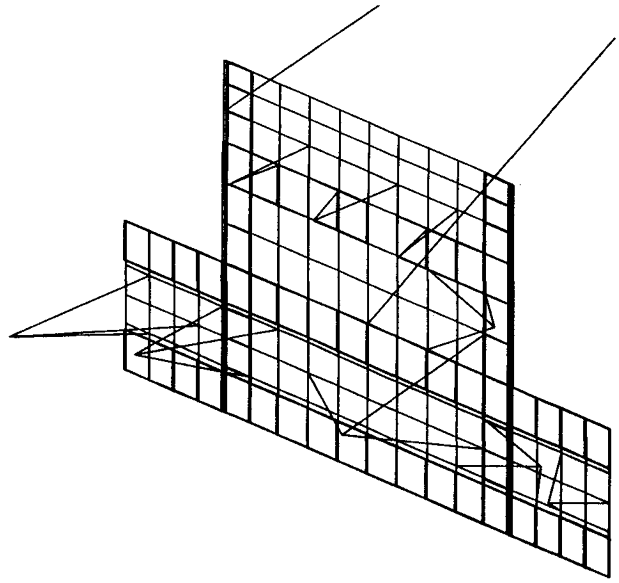

Fig. 5 Platform finite element model.

In this approach, no major computations are required since the gradients of the strain energy ratios are already computed for use in the design optimization process. Also, since this new approach takes into account both the sensitivity coefficients of strain energy ratio with respect to shear modulus and the material test data, it is reasonable to expect the adjusted modal loss factors to match better with those obtained from a complete analysis using a G value corresponding to the appropriate frequency.

Comparison of the Two Approaches on the Payload Mounting Platform

The correction-factor approach of Ref. 2 and the new approach using function sensitivities are compared on the precision mounting platform to assess the accuracy of these different methods. The viscoelastic material used for the damping layer is SMRD 100F90B, and the reduced nomogram for the same material is shown in Fig. 1. The shear modulus and material loss factor of this material, at several frequencies, are provided in Table 1. The frequencies correspond to those of the precision mounting platform.

The adjusted modal loss factors, obtained from the two different methods, are compared in Table 2. The exact values of the modal loss factors, obtained from performing six

Table 1 Material SMRD 100F90B test results

Mode i	w_i, Hz	Temperature, °F	Shear modulus, $G(w_i)$	Material loss factor, β_i
1	15.65	67	1340	0.8198
2	23.16	67	1698	0.8177
3	46.96	67	2654	0.7836
4	50.09	67	2767	0.7788
5	62.98	67	3212	0.7594
6	76.02	67	3635	0.7411
7	91.88	67	4119	0.7209

Table 2 Comparison of modal loss factor [$G_{\mathrm{ref}} = G(\omega_1)$]

Mode i	w_i, Hz	Modal loss factor [from analysis with $G(w_i)$]	Modal loss factor [using correction factor $\sqrt{G(f_r)/G_{\mathrm{ref}}}$]	Modal loss factor [new method at G_{ref}]
1	15.65	0.0758	0.0758	0.0758
2	23.16	0.0816	0.0974	0.0829
3	46.96	0.0786	0.1642	0.097
4	50.09	0.0760	0.1093	0.0744
5	62.98	0.0445	0.0862	0.0506
6	76.02	0.0593	0.1154	0.0667
7	91.88	0.0502	0.1358	0.065

separate normal modes analyses with each run having a different shear moduls $G(\omega_i)$, $i = 2, \ldots, 7$, are shown in the third column. Clearly, the proposed approach accounts for the frequency dependence of the damping material properties more accurately.

Damping Optimization Problem

The damping optimization problem can be stated as a nonlinear programming problem[7] of the following form:
Find the vector of design variables X that

Minimize

$$F(X) \qquad \text{(objective function)}$$

subject to

$$g_j(X) \leq 0, \qquad j = 1, m \qquad \text{(inequality constraints)}$$

$$x_i^l \leq x_i \leq x_i^u \qquad \text{(side constraints)} \qquad (13)$$

For the damping design problem, the design variables include the 1) constraining layer outer metal matrix composite (MMC) facesheet thickness, 2) constraining layer honeycomb core thickness, 3) viscoelastic material (VEM) layer thickness, 4) viscoelastic material shear modulus, 5) base outer MMC thickness, and 6) base honeycomb core thickness. A diagram illustrating these design variables in the three-layered sandwich structure is shown in Fig. 3. The inner MMC facesheet thickness of the base and constraining layers are not treated as variables for the optimization process. Inequality constraints $g_j(X)$ are imposed on the frequency requirements of the structure and damping levels. Manufacturing requirements in the form of bounds on the design variables (thicknesses of MMC facesheets and VEM layer) are imposed.

The damping design problem, stated in Eq. (13), is solved using the DAMP-OPT damping design optimization program. The program architecture is shown in Fig. 4. DAMP-OPT is a modified version of ODAMP[8] program that uses MSC/Nastran Version 66 for normal modes analysis and strain energy calculations and ADS[9] program for numerical optimization. ADS contains a wide variety of gradient-based numerical search algorithms and in this paper the method of feasible directions is used. In this method, only the set of active constraints is used for calculating the usable-feasible search direction.[7]

Approximate Problem Generation

The nonlinear programming problem, stated in Eq. (13), is replaced by a sequence of approximate optimization problems in which the objective and constraint functions are approximated as functions of the design variables. Since the approximate models for the constraint and the objective functions essentially use the first-order Taylor series expansion, their evaluations require substantially less computational effort.[10] The hybrid approximation,[11] which is more conservative than the linear and reciprocal approximations, is used for the constraint functions:

$$\tilde{g}(X) = g(X_0) + \sum_{i=1}^{n} B_i(x_i - x_{0i})(\partial g/\partial x_i)_{x_0}$$

where

$$B_i = 1 \qquad \text{if} \quad x_{0i}^* \partial g/\partial x_i > 0$$

$$= x_{0i}/x_i \qquad \text{if} \quad x_{0i}^* \partial g/\partial x_i \leq 0 \qquad (14)$$

When $B_i = 1$, the hybrid approximation is the same as direct Taylor series approximation and otherwise reduces to the reciprocal approximation.

The eigenvalue λ_r, eigenvector ϕ_r, and strain energy ratio ϵ^r sensitivities that are required for the constraint sensitivities are computed as follows:

$$\frac{\partial \lambda_r}{\partial x_i} = \frac{\phi_r^T(\partial K/\partial x_i - \lambda_r \partial M/\partial x_i)\phi_r}{\phi_r^T M \phi_r} \qquad (15)$$

$$\frac{\partial \epsilon^r}{\partial x_i} = \frac{V^r(\partial V_v^r/\partial x_i) - V_v^r(\partial V^r/\partial x_i)}{V^{r2}} \qquad (16)$$

Details of the derivation for strain energy ratio sensitivity can be found in Ref. 8. The Nelson method[12] is used for eigenvector sensitivities.

The approximate optimization problem is now stated as

Minimize

$$\tilde{F}(X)$$

subject to

$$\tilde{g}_j(X) \leq 0$$

$$x_i^l \leq x_i \leq x_i^u \qquad (17)$$

The approximate optimization problem is solved using the method of feasible directions, programmed in the ADS optimizer.

Vibration Mode Tracking

In structural synthesis, the vibration modes are identified by their order based on the magnitude. During the design optimization process, it is possible that these modes may switch locations due to changes in the stiffness-to-mass ratios. In other words, after a few design iterations, a bending mode

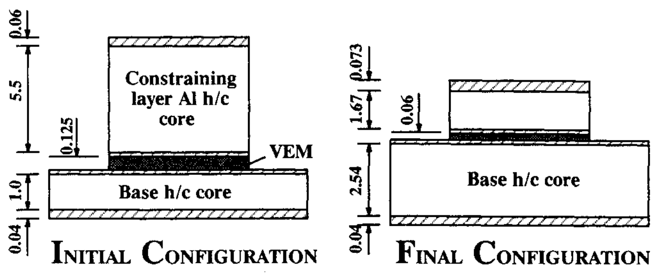

Fig. 6 External stiffener—initial and final cross sections.

which occupied the third position at the initial design may no longer be the third vibration mode. Instead the third mode could now be a torsional mode and the bending mode of interest could now be the fifth mode. If this modal switching is not tracked, the optimization process would be constraining the wrong modes.

The procedure that is used to track the vibration modes is based on the modal orthogonality relation,

$$\phi_r^T M \phi_r = 1$$

$$\phi_s^T M \phi_r = 0, \qquad r \neq s \tag{18}$$

The actual implementation of the mode-tracking capability is done using the MSC/Nastran's DMAP (direct matrix abstraction programming) language. After each complete normal modes analysis, the scalar products ψ_s of the current design eigenvectors ϕ and the previous design eigenvectors ϕ' are computed. Based on modal orthogonality, only one value of ψ_s should be high compared to the rest, thereby identifying the position of the mode of interest.

$$\psi_s = \phi_r^T \phi_s', \qquad s = 1,2,\ldots,R \tag{19}$$

Design Example: Payload Mounting Platform—External Stiffener Concept

The payload mounting platform (PMP)[6] is used as the design example for the damping optimization outlined in this paper. Figure 2 shows the PMP, and the finite element model is shown in Fig. 5. The platform is kinematically mounted to the satellite primary structure at four points and supports an instrument weight of 300 lb. The design, shown in Fig. 2, is for an external stiffener configuration where strips of viscoelastic material and aluminum honeycomb core with MMC facesheets are bonded to the base panel to provide damping and added stiffness.

The design optimization problem is to minimize the weight of the platform subject to a 15.5-Hz minimum frequency requirement and at least a 10% composite loss factor in all modes less than 100 Hz. The 15.5-Hz fundamental frequency requirement is to avoid dynamic coupling with other spacecraft subsystem frequencies. A feasible design of about 30% lesser weight than the initial design is obtained in six design iterations. For this design, a material loss factor β^r of 1.0 was assumed for all of the modes. Significant changes were made to the VEM damping layer thickness, the constraining layer, and the base layer thickness to obtain the final design. The initial and final cross sections are shown in Fig. 6. The weight iteration history and damping-level iteration histories for the first five modes are shown in Figs. 7 and 8, respectively.

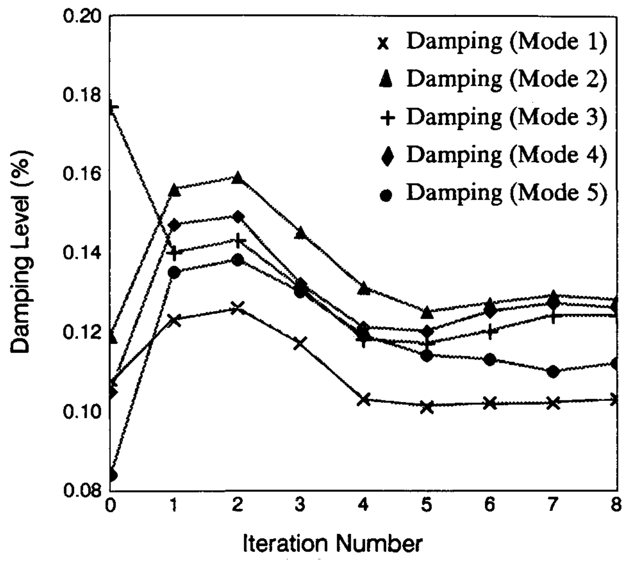

Fig. 8 Damping iteration history.

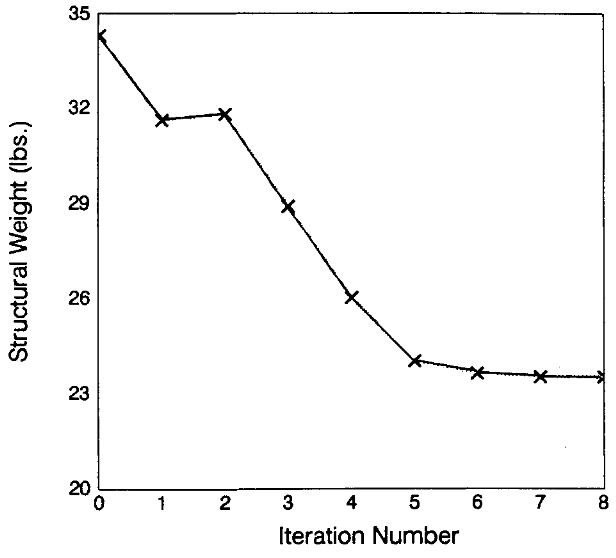

Fig. 7 Weight iteration history.

Conclusions

Passive damping, using viscoelastic materials, is a robust method of providing moderate levels of vibration control in large space structures. This paper provides an efficient methodology for the design optimization of viscoelastically damped honeycomb structures, for passive vibration control. Hybrid approximations to the design objective and constraint functions are used to reduce the computational effort required during the iterative optimization process. These approximations for strain energy ratios and eigenvalues are sufficiently accurate to overcome damping and frequency constraint violations quickly. Also, it is essential to track and detect any switching of modes when considering several different vibration modes in the design process.

A new approach to characterize the viscoelastic material property variations with frequency is outlined. Since this approach takes into account both the strain energy ratio sensitivity with respect to the material shear modulus and the viscoelastic material test data from the reduced-frequency nomogram, it is reasonable to expect a better approximation of the material property variations with frequency. This procedure is implemented in the damping design architecture.

Acknowledgments

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